Triaxial Accelerometers for Improved Seismic and Geodetic Measurements

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“The standard by which other standards are measured”

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Abstract:

Triaxial Accelerometers have been developed with a full-scale range of several G's, parts-per-billion sensitivity and good long-term stability. The high range eliminates clipping (saturation) and sensitive measurements of tilt can be made without a leveling system. An internal alignment matrix uses the invariance of the Earth's gravity vector as a reference to measure seismic events and resultant earth movements. Improved, high-resolution measurements of earthquakes, tilts, slow-slip and long-term events are possible for phenomena occurring over a time spectrum from fractions of a second to years.

Background:

Traditional broadband seismometers and tiltmeters do not have the range to measure strong seismic events and traditional strong motion sensors do not have the sensitivity or stability to make good long-term geodetic measurements. Quartz Crystal Triaxial Accelerometers, with parts-per-billion resolution, can measure strong earthquakes without clipping and can use the invariance of Earth's 1 G gravity vector as a reference for long-term measurements.

Construction, Operation and Performance of Nano-Resolution Accelerometers:

Quartz crystal resonators (Figure 1) convert analog force inputs to digital outputs with parts-per-billion resolution:
http://www.paroscientific.com/pdf/G8086_Noise_Floor_of_Quartz_Crystal_Sensors.pdf).

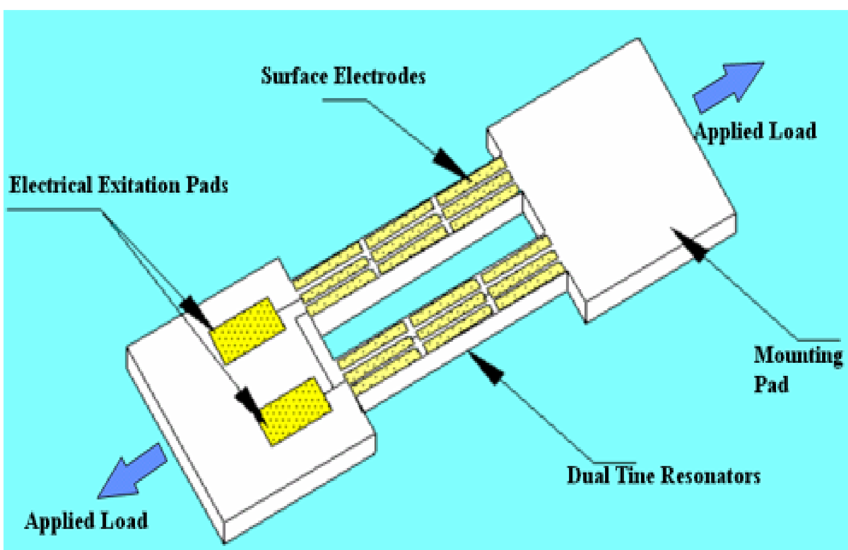

Figure 1

The seismic sensors employ suspended masses as the acceleration-to-force generators (References 1-3). Advanced electronics and frequency counting algorithms enable these inherently-digital sensors to make measurements at about -180dB relative to full scale range.

Figure 2 shows the experimental power spectral density of an isolated quartz resonator. The 5-stage, low-pass (anti-aliasing) IIR (Infinite Impulse Response) filters attenuate at 100 dB/decade above the selected corner frequency.

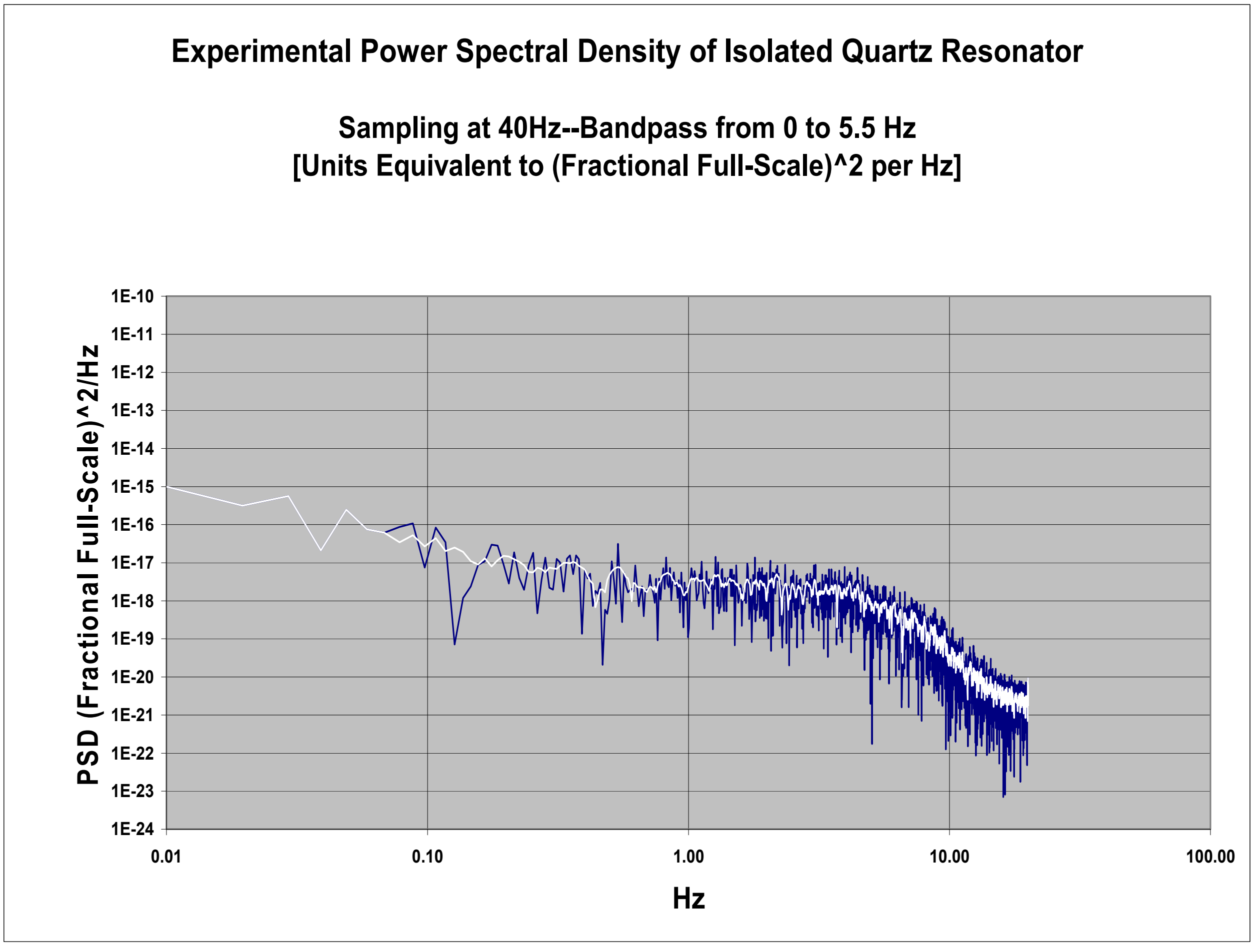

Figure 2

The Nano-Resolution Accelerometers are individually calibrated and tested. Coefficients are derived to temperature compensate and linearize the outputs of each accelerometer. Three accelerometers are oriented in nominally orthogonal X, Y and Z positions in a triaxial package. A 24 position tumble test is performed in Earth's 1G gravity field and an alignment matrix is derived to mathematically align the internal orthogonal axes to Cartesian coordinates (See Appendix). This allows the accurate measurements of tilts on each axis without a leveling system. By taking the square root of the sum of the squares (RSS) of the orthogonal acceleration values, Earth's 1G gravity vector can be computed. In the absence of earthquakes, the gravity vector is invariant over time and any changes in the gravity vector can be associated with long-term drift of the accelerometers. In-situ calibration methods can compensate for long-term sensor drift (See: http://www.paroscientific.com/pdf/G8097_Calibration_Methods_to_Eliminate_Sensor_Drift.pdf) and Reference 4.

Thus high-resolution measurements of earthquakes, tilts, slow-slip and long-term events are possible for phenomena occurring over a time spectrum from fractions of a second to years.

Experimental Results and Analysis:

The triaxial package of Nano-Resolution Accelerometers and a data logger were incorporated into an Acceleration Ocean Bottom Seismometer (AOBS) (Reference 5). The AOBS was deployed near the epicenter of the 2011 Tohoku-Oki earthquake from May 2013 to March 2014. Located 35 meters away were a traditional broadband ocean bottom seismometer, differential pressure gauge and ocean bottom tiltmeter. Because the range of the Triaxial Accelerometer is greater than +/- 2 G's on all axes, earthquakes measured with the AOBS never clipped whereas the comparison broadband seismometer and tiltmeters saturated with moderate seismic inputs.

The Nano-Resolution Quartz Resonator Technology allows measurements to parts-per-billion of full scale. Figure 3 is a Power Spectral Density (PSD) plot of the measured gravity vector showing the microseismic peak and noise floor.

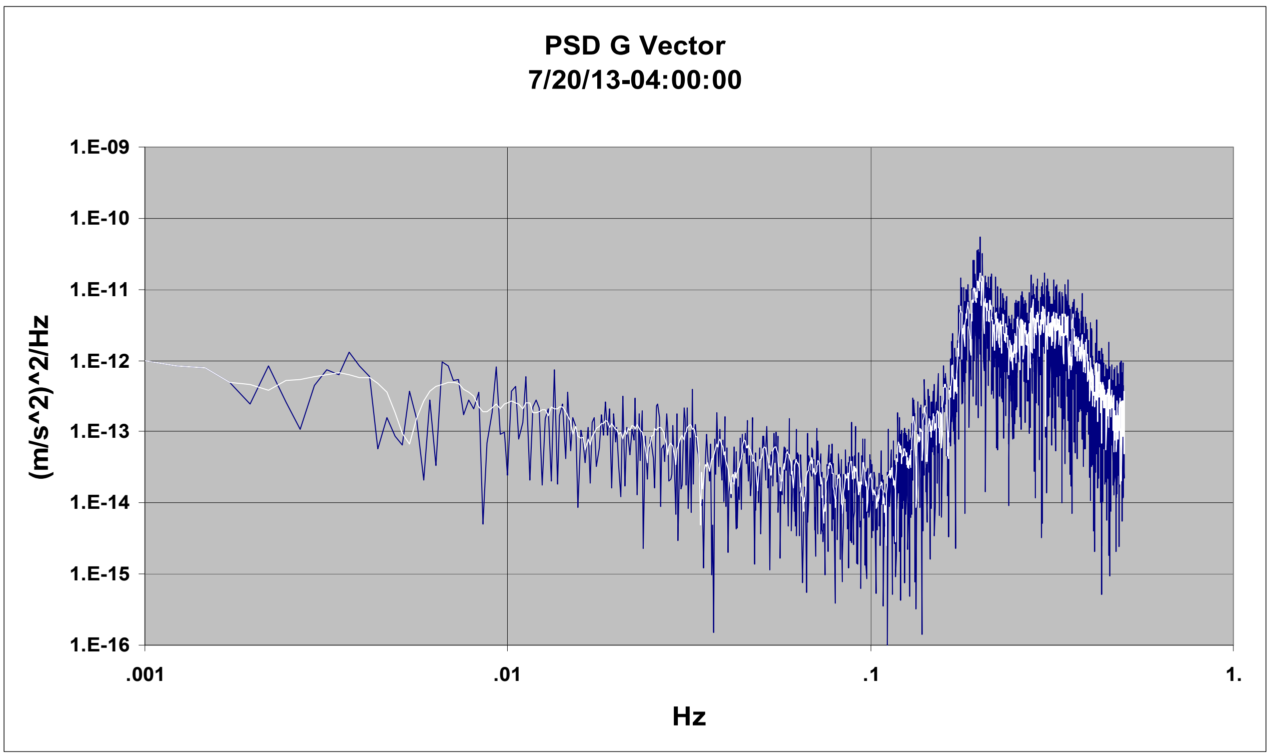

Figure 3

Using the alignment matrix allows accurate measurements of seismic signals and tilts on decoupled orthogonal axes. By taking the square root of the sum of the squares (RSS) of the orthogonal acceleration values, the gravity vector can be computed and compared to the static value of Earth's "invariant" 1G gravity value at the deployment site.

The importance of using the aligned values of measured accelerations is illustrated in Figure 4 where earthquake signals are plotted with and without the alignment matrix. Without the alignment matrix (red), there appears to be a shift in the measured G vector. The aligned measurement (blue) is invariant before and after the earthquake and a "true" measure of acceleration and tilt is obtained on the orthogonal axes.

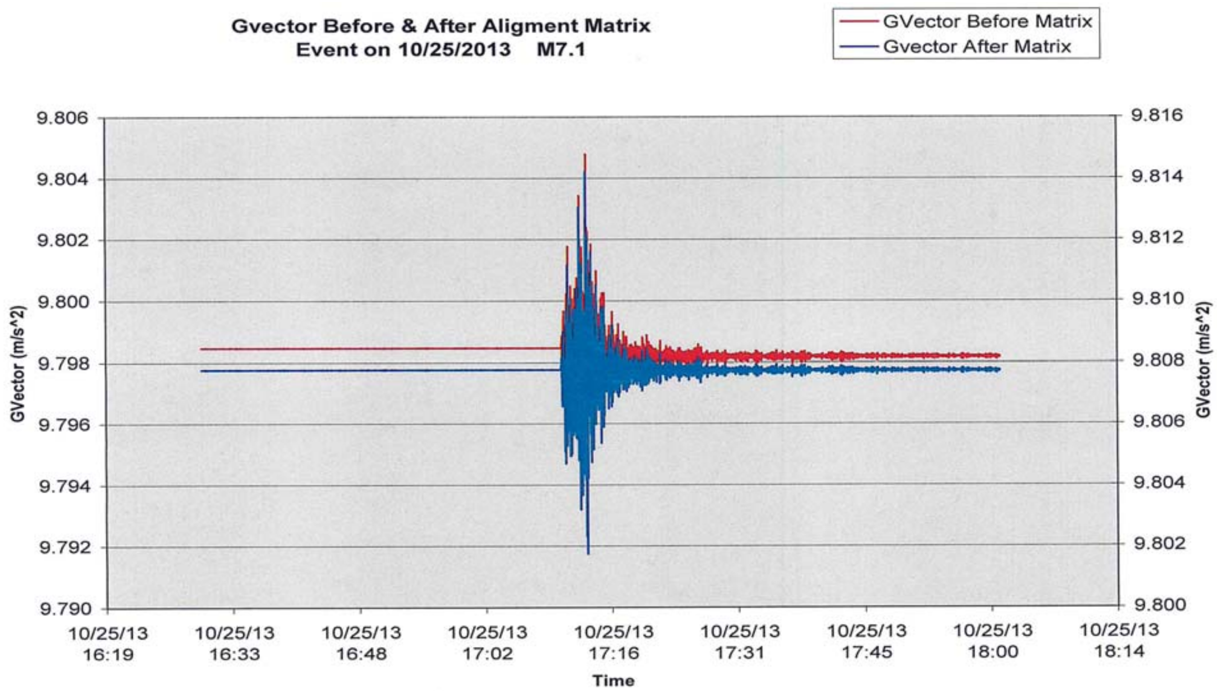

Figure 4

Figure 5 plots the long-term drift of the measured gravity vector. The residuals of a mathematical fit are a few parts-per-million (ppm) of the 20 m/sec^2 full scale range. An in-situ calibration method has been developed (Reference 4).

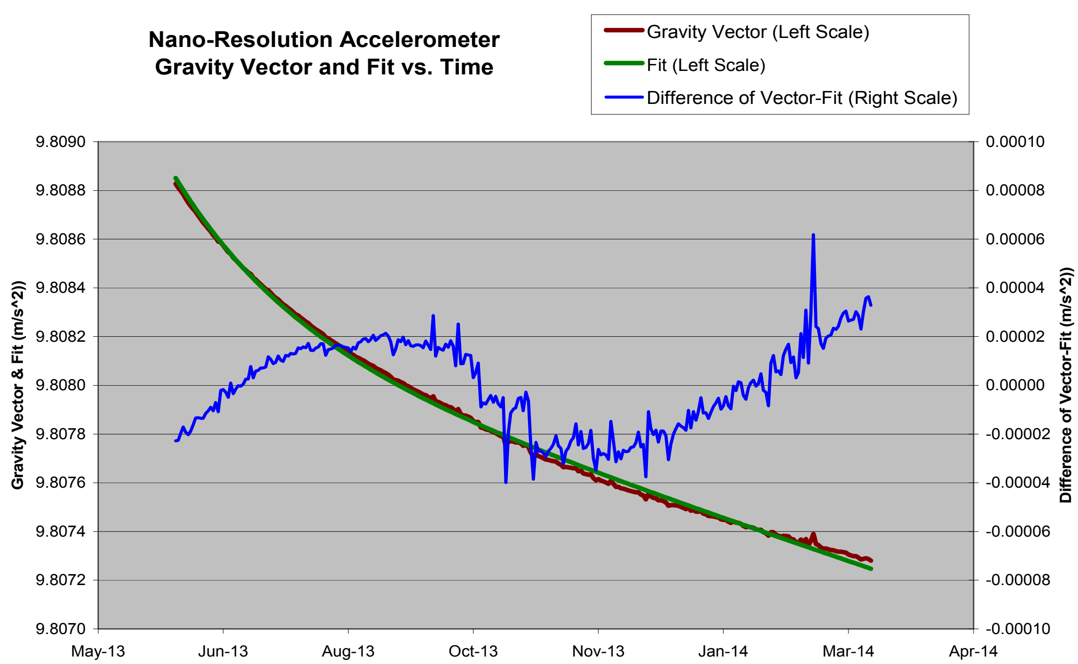

Figure 5

Reference 6 describes a new tool to measure Acceleration, Pressure & Temperature (APT). Seismic and tilt measurements made with the Quartz Triaxial Accelerometer are compared to a broadband seismometer in Figure 6.

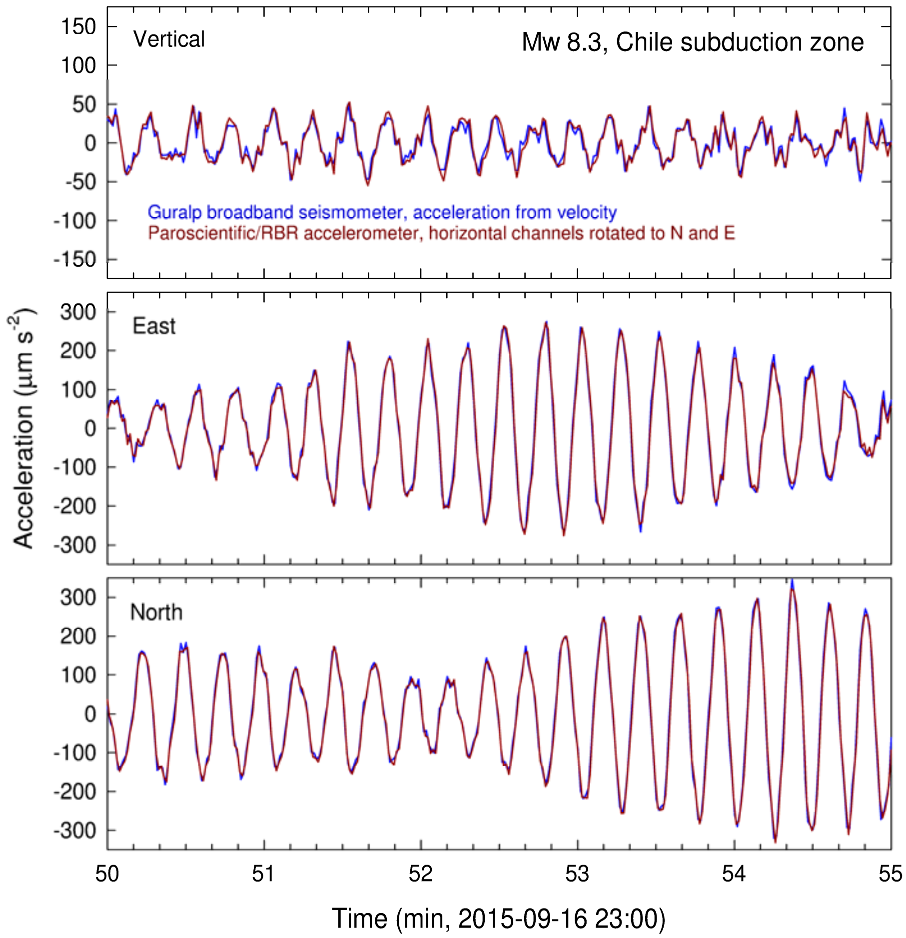

Figure 6

Calculation of Horizontal Tilts Referenced to Earth's Gravity Plumb Line

The triaxial accelerometer is calibrated with an internal alignment matrix to ensure the three axes are orthogonal to each other (See Appendix). These internal axes are labeled X, Y, Z according to the convention shown in Figure 1 of the Appendix. Nano-resolution processing electronics from Paroscientific and Quartz Seismic Sensors provide linear, temperature-compensated, **aligned**, values of X, Y, Z and total Vector accelerations. Measurements of earth movements from co-seismic events and the passage of frontal bores are described in Reference 5. The plots in Figure 7 show tilt measurements due to tidal loading and nearby gas venting (Reference 6).

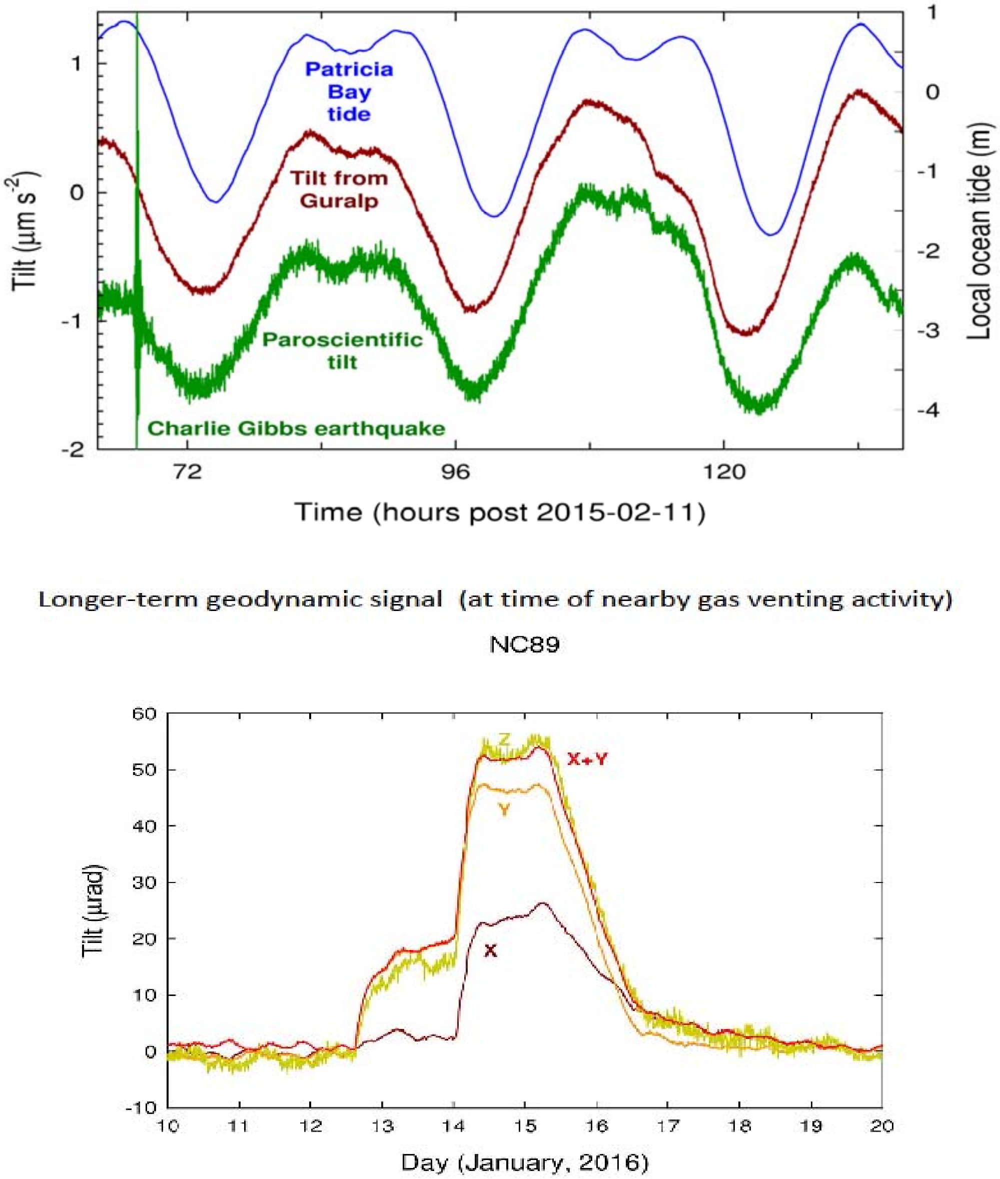

Figure 7

There is a different set of external axes that relates the internally aligned axes of the Triaxial Accelerometer to the orientation on the platform on which it is mounted. The external platform axes can be defined by geometry and a compass. Tilting of the platform changes the values of the components of Earth's gravity vector on all 3 axes. The acceleration outputs on the horizontal axes are the 1G vector multiplied by the sine of the angle between the vertical component of the horizontal axes of the sensor and the external Cartesian axes referenced to the seafloor. The acceleration output on the vertical axis is the +1G vector multiplied by the cosine of the angle between the vertical axis of the sensor and the true +1G plumb line. AX, AY, and AZ are the acceleration outputs from each orthogonal axis and V is the local gravity vector, calculated as $V = \sqrt{(AX^2 + AY^2 + AZ^2)}$. The initial angles of the platform for X (Horizontal East), Y (Horizontal North), and Z (Vertical Up) are calculated from the components of the G vector, V, along X, Y, and Z:

$AX = V \sin(\theta_x) = V\sin$ [Initial Angle for X (Horizontal East)]
$AY = V \sin(\theta_y) = V\sin$ [Initial Angle for Y (Horizontal North)]
$AZ = V \cos(\theta_z) = V\cos$ [Initial Angle for Z (Vertical Up)]

One way to calculate the relative changes (tilts = $\Delta\theta$) in each axis is to differentiate the above:

$\Delta AX = [V\cos(\theta_x)]\, \Delta\theta_x$ Tilt for X (Horizontal East) $= \Delta\theta_x = \Delta AX/(V\cos(\theta_x))$
$\Delta AY = [V\cos(\theta_y)]\, \Delta\theta_y$ Tilt for Y (Horizontal North) $= \Delta\theta_y = \Delta AY/(V\cos(\theta_y))$
$\Delta AZ = -[V\sin(\theta_z)]\, \Delta\theta_z$ Tilt for Z (Vertical Up) $= \Delta\theta_z = -\Delta AZ/(V\sin(\theta_z))$

Example: The readings for a Triaxial Accelerometer deployed on the seafloor were:

Date	AX [m/s^2]	AY [m/s^2]	AZ [m/s^2]	Vector [m/s^2]
Start 7/1/17	0.50386	- 0.49723	9.77445	9.80006
End 5/1/18	0.46153	- 0.51759	9.77549 (Drift Corrected)	9.80006

The starting angles and subsequent tilts for X, Y, & Z are calculated from the above equations.

The starting angles of each axis and the changes in acceleration (End – Start) are:

$\theta_x = \sin^{-1}(AX/V) =$ **0.05144 rad** $\Delta AX =$ **- 0.04233 m/s²**
$\theta_y = \sin^{-1}(AY/V) =$ **- 0.05076 rad** $\Delta AY =$ **- 0.02036 m/s²**
$\theta_z = \cos^{-1}(AZ/V) =$ **0.07230 rad** $\Delta AZ =$ **0.00104 m/s²**

The tilts for each axis are:

$\Delta\theta_x = \Delta AX/(V\cos(\theta_x)) = -0.04233/(9.80006*\cos(0.05144)) =$ **- 0.00432 rad**
$\Delta\theta_y = \Delta AY/(V\cos(\theta_y)) = -0.02036/(9.80006*\cos(-0.05076)) =$ **- 0.00208 rad**
$\Delta\theta_z = -\Delta AZ/(V\sin(\theta_z)) = 0.00104/(9.80006*\sin(0.07231)) =$ **- 0.00147 rad**

The angle of the horizontal X-Y plane = $\theta_{HP} = \sqrt{(\theta_x^2 + \theta_y^2)}$.

The angles at the start are: θ_x (start) = 0.05144 rad and θ_y (start) = - 0.05076 rad.

The angles at the end equal the starting angles plus the subsequent tilts:
$\theta_x + \Delta\theta_x = 0.04721$ and $\theta_y + \Delta\theta_y = -0.05284$

The tilt of the X-Y horizontal plane, $\Delta\theta_{HP}$, relative to the plumb line gravity vector is calculated as the angle of the plane at the end minus the angle of the plane at the start:

$\Delta\theta_{HP} = \sqrt{((\theta_x + \Delta\theta_x)^2 + (\theta_y + \Delta\theta_y)^2)} - \sqrt{(\theta_x^2 + \theta_y^2)} = 0.07079 \text{ rad} - 0.07226 \text{ rad} =$ **- 0.00147 rad**

	X (rad)	Y (rad)	Z (rad)	Horizontal Plane (rad)
Starting Angle (θ) on 7/1/17	0.05144	- 0.05076	0.07230	0.07226
Overall tilt (Δθ) from 7/1/17 to 5/1/18	- 0.00432	- 0.00208	- 0.00147	- 0.00147

Strap-down Gravimeter Tests

Tests were run on a Quartz Crystal Triaxial Accelerometer to determine its suitability for use as a strap-down gravimeter for AUV gravity surveys. The performance goal was to measure gravity to better than 1 mGal/day with a +/- 3G full-scale triaxial accelerometer. The outputs from the Nano-Resolution processing electronics were aligned values of X, Y, Z, and Total Vector accelerations as well as sensor temperature. The IIR filter was set at IA =13 (0.16 Hz corner frequency). Data was logged at 1 Hz and analyzed at 1 minute intervals with 60 seconds averages. Figure 7 shows the measured gravity vector and sensor temperature versus date/time. A linear correction was made for residual thermal errors caused by up to 2 degrees C variation. A linear correction was made for daily drift.

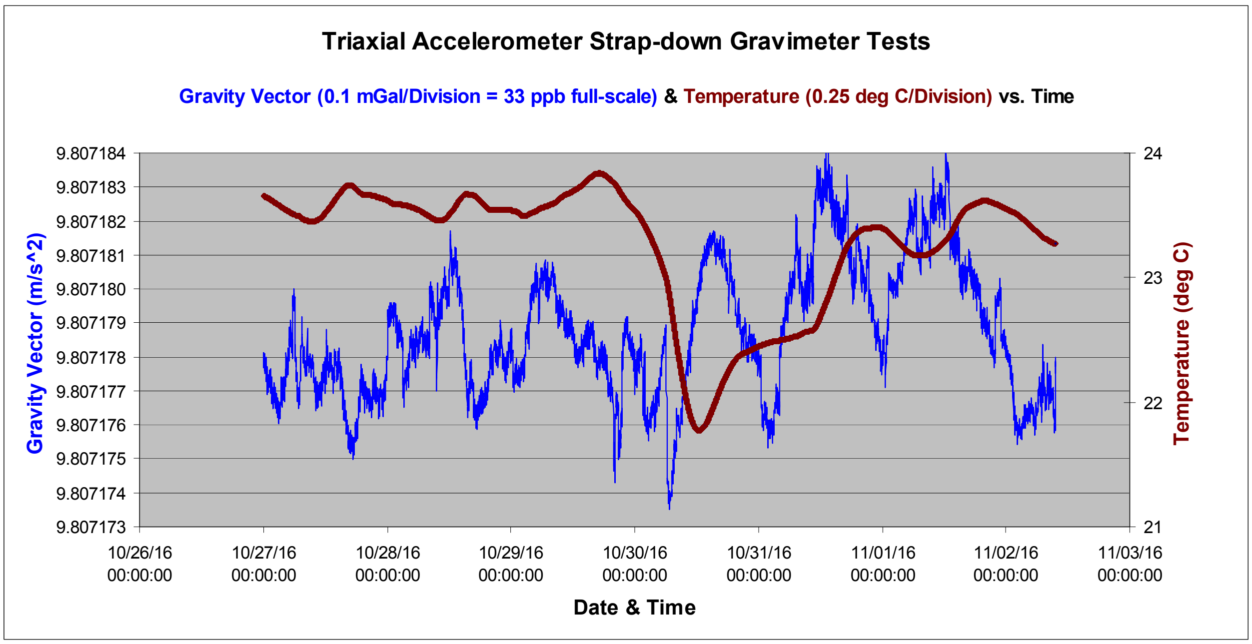

Figure 8

Conclusions:

Triaxial Accelerometers have been developed with a full-scale range of several G's, parts-per-billion sensitivity and good long-term stability. The high range eliminates clipping (saturation) and sensitive measurements of tilt can be made without a leveling system. An internal alignment matrix uses the invariance of the Earth's gravity vector as a reference to measure seismic events and resultant earth movements. The Triaxial Accelerometers play a key role in sensor modules that have been developed for both disaster warning and geodetic measurements. The instrumentation package includes deep-sea absolute pressure gauges, a triaxial accelerometer, and nano-resolution processing electronics. An in-situ calibration system eliminates pressure sensor drift. In addition to making the seismic and tilt measurements, the Triaxial Accelerometer outputs are used to eliminate the orientation sensitivity of the pressure gauges. The SOS modules work on cabled systems (both internally and externally mounted), in remote ocean bottom recorders, and in underwater vehicles (See: http://www.paroscientific.com/pdf/P20_Seismic_Oceanic_Sensors_(SOS).pdf).

Acknowledgements:

We would like to acknowledge the excellent work of Y. Fukao, H. Sugioka, A. Ito, and H. Shiobara for developing new Acceleration Ocean Bottom Seismometers (AOBS) and E.E. Davis, J. Paros, G. Johnson, M. Heesemann, and R. Meldrum for developing new Acceleration, Pressure & Temperature (APT) tools that use the Quartz Triaxial Accelerometers described in this paper.

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[4] J.M. Paros, "Triaxial Acceleration Assembly and In-situ Calibration Method for Improved Geodetic and Seismic Measurements", U.S. Patent 9,645,267 B2, May 9, 2017

[5] Fukao, Y., H. Sugioka, A. Ito, H. Shiobara, J. M. Paros, and R. Furue, "Sensing of upslope passages of frontal bores across the trench slope break of the Japan Trench", (2016), J. Geophys. Res. Oceans, 121, doi:10.1002/2015JC011432.

[6] Earl E. Davis, Martin Heesemann, Joseph J. Farrugia, Greg Johnson, Jerome Paros; APT: An Instrument for Monitoring Seafloor Acceleration, Pressure, and Temperature with Large Dynamic Range and Bandwidth. *Bulletin of the Seismological Society of America (Feb 2019)*; 109 (1): 448–462. doi: https://doi.org/10.1785/0120180132.

Appendix

Aligning the Triaxial Accelerometer to Cartesian Coordinates

Abstract:

The Quartz Triaxial Accelerometer consists of three single-axis accelerometers mounted in a triaxial arrangement such that each component is near an orthogonal x, y, z axis. After factory calibration, an alignment matrix is calculated that provides the outputs along true orthogonal Cartesian axes with outputs ax, ay, az (See Figure 1). As a result, the magnitude of the total g vector is constant in any placement of the triax. If the triax were positioned such that ax=0 and ay=0 (horizontal axes), az (vertical axis) would be exactly the total g value (points directly along the gravitational pull). In most deployments, and absent an elaborate positioning apparatus, the outputs are not exactly along cardinal directions (up/down, east/west, north/south). The outputs can be rotated with matrix algebra to derive outputs into any other orthogonal frame. Here we use the aviation language of pitch, roll, and yaw to describe the method and how to calculate the three angles of rotation. A proper external alignment rotation will eliminate the cross-axis sensitivity both statically (tilts under gravity) and dynamically (i.e. vibrations along cardinal directions).

Background:

Traditional analog seismometers came in two varieties: horizontal and vertical. The latter was more difficult to construct and often less sensitive, or with a different frequency response. To make them equal, Streckeisen introduced the STS-2 broadband seismometer with three identical components that were positioned in a triad with inclination of 54.7 degrees each (Reference Havslov, p. 69). The three outputs are then rotated electronically and recombined with a 3x3 matrix to x, y, z components.

Similarly, the Quartz Triaxial Accelerometer consists of three identical single-axis accelerometers with the same scale factor and frequency response, arranged in a nearly, but not exactly, orthogonal triad. Since the full scale exceeds earth's gravity, it can be placed in any orientation without a leveling system. A proper 3-dimensional matrix rotation recombines the outputs with a digital data processor into x, y, z components.

Internal and External Alignment Matrices:

There are two matrix manipulations necessary to arrive at the final result. The first matrix is calculated by the manufacturer to provide outputs ax, ay, az that are orthogonal. This is referred to as an "internal" alignment matrix. The internal alignment does not guarantee that if the triax is placed on a level surface that the horizontal axes are exactly zero (the typical internal alignment is within about 1 degree or less).

For the external alignment, the triax could be physically positioned to be exactly horizontal. This could be accomplished with two leveling screws that are turned until both ax and ay are exactly zero. If the nominal z direction is up, the az component will then show the full g magnitude as it is exactly aligned with the vertical direction (as defined by the g vector). Leveling in the horizontal direction is accomplished by adjusting two small angles (called pitch and roll below) with the net result that the polar angle of the az direction is adjusted to zero.

That leaves the azimuthal adjustment. For illustration, it is assumed that the ax component is pointing towards east and the ay component towards north (but that's only a convention). The ax, ay outputs are not exactly aligned with cardinal directions (typically off by 1 deg or less) and the g vector is of no help. Now we need a known vibration in the horizontal plane such that we can physically turn the sensor in the azimuth until it is aligned. This could be accomplished with a precisely aligned x/y shake table. An easier method involves using a 90 degrees bracket and rotating the sensor 90 degrees in the x-y plane to place ay near vertical. The sensor could then be physically rotated in the x-y plane until ax=0 and repositioned horizontally.

Roll and Pitch Correction in Matrix Form:

A three-dimensional rotation can be specified in a number of ways. Generally it involves three Euler angles that are rotated one at a time while holding the other two fixed. Because of different conventions, the sign (or direction) of angles must be carefully checked. Here we are using common terms from avionics. The pitch, roll, and yaw angles are illustrated in Figure 1.

Here we define the z direction as vertical (up) along the gravitational direction. Note that even though gravity pulls down (exerts a force on the proof mass of the accelerometer), the sensor frame accelerates upwards and the az output is positive (near 9.8 m/s/s). If there is added acceleration in the z direction, then az increases. This is best visualized in an elevator – our weight increases as we accelerate upwards.

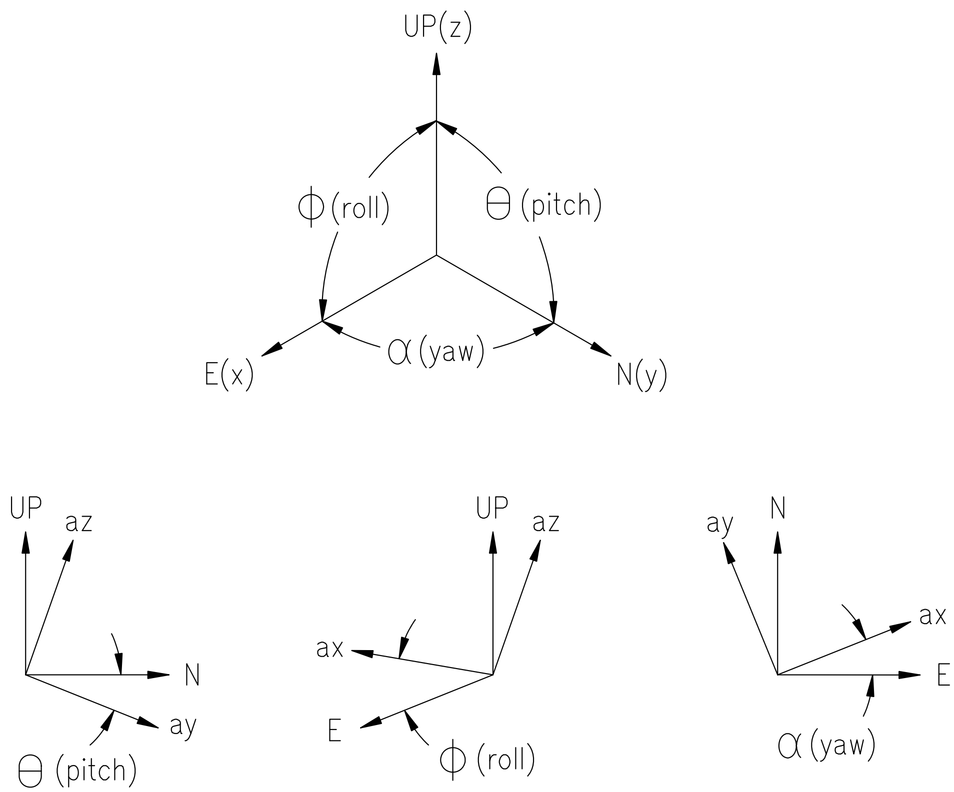

Figure 1

The nominal horizontal directions in a right-handed frame are x along the east (E) direction and y along the north (N direction). Here we define the pitch along the y direction. The pitch angle θ is defined in the y-z plane (rotation about the x axis). If ay is positive the pitch angle is also positive (upwards). In the y-z plane, the rotation matrix is calculated from the pitch angle θ:

The new vector components are

$ay' = ay \cos\theta - az \sin\theta$

$az' = ay \sin\theta + az \cos\theta$

The goal is to align the pitch to zero. Setting ay'=0 gives: $ay \cos\theta = az \sin\theta$ and $\tan\theta = ay/az$

Hence the pitch angle θ is the arctangent of the ratio ay/az. This ratio is simply the static output of the accelerometer. As an example: If the measured outputs (in earth's gravity) are az= 9.7965 m/s/s, ay=-0.1234 m/s/s, then θ=-0.7217 deg. (pitch is negative - down in the north direction).

Next we define the roll in analogous fashion in the x-z plane (rotation about the y axis). The roll angle α is positive if the ax output is positive (upwards in the east direction). The rotation matrix for this second operation is:

$ax' = ax \cos\phi - az \sin\phi$

$az' = ax \sin\phi + az \sin\phi$

Again, the goal is ax'= 0 (no roll), hence $\tan\phi = ax/az$ and the roll angle ϕ is the arctangent of the measured ratio ax/az.

In three dimensions, the pitch rotation matrix is

1	0	0
0	C1	-S1
0	S1	C1

Where C1 = cos θ, S1 = sin θ

The roll rotation matrix is

C2	0	-S2
0	1	0
S2	0	C2

Where C2 = cos ϕ, S2 = sin ϕ

The combined pitch and roll rotation is the product of the two matrices:

C2	-S1S2	-C1S2
0	C1	-S1
S2	S1C2	C1C2

The new acceleration vector output aligned with the true horizontal plane is the measured output vector components multiplied by the combined matrix. The horizontal alignment will eliminate any cross-axis errors in the ax' and ay' outputs if the vibration is truly vertical.

Yaw Correction in Matrix Form:

We define the yaw angle α as a misalignment of the ax (east) output in the direction of ay (north) (rotation in the x–y plane about the z axis). This is not easily determined except with a known directional input in the horizontal plane. This could be determined on an x/y shake table or with an earthquake signal from a known location. As shown above, it could also be measured by rotating the ay axis into the vertical direction and measuring the ratio ax/ay in a static field (prior to deployment).

In a deployed horizontal direction, the only signal stems from an external vibration with (measured) amplitude Δax and Δay. As before, the two-dimensional rotation in the horizontal plane is

$\Delta ax' = \Delta ax \cos\alpha - \Delta ay \sin\alpha$ and $\Delta ay' = \Delta ax \cos\alpha + \Delta ay \sin\alpha$

Again, the goal is $\Delta ax' = 0$ (no cross-axis sensitivity if the vibration is along ay) and we find the yaw misalignment as:

$\tan\alpha = \Delta ax/\Delta ay$

The yaw correction in a three-dimensional matrix (where C3 = cos α and S3 = sin α) is:

$$\begin{matrix} C3 & -S3 & 0 \\ S3 & C3 & 0 \\ 0 & 0 & 1 \end{matrix}$$

The yaw correction can be combined with the pitch-and-roll correction into a single 3X3 matrix:

C2C3	**(-C1S3 – S1S2C3)**	**(-C1S2C3 + S1S3)**			**1**	**-α**	**-ϕ**
C2S3	**(C1C3 – S1S2S3)**	**(-S1C3 – C1S2C3)**		For Small Angles (radians) »»»»»»»	**α**	**1**	**-θ**
S2	**S1C2**	**C1C2**			**ϕ**	**θ**	**1**

The somewhat uneven look of the matrix elements is a consequence of the fact that three-dimensional rotations are performed in three steps that are not commutative. But, if properly calculated, the matrix constitutes a proper rotation with a determinant of 1. The magnitude of the g vector is unchanged under proper rotations and the matrix is valid at all angles even if they are far from cardinal directions.

Appendix References:

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